Feasibility Conditions for Interference Neutralization in Relay-Aided Interference Channel

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***Abstract*—**

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***Index Terms*—**

I. Introduction

ULTI-INPUT-MULTI-OUTPUT (MIMO) interference channel (IC) and interference broadcast channel (IBC) refer to setups in cellular networks where multiple multi-antenna base stations (BSs) transmit to multiple multi-antenna users in the same time/frequency resource without data sharing among the BS. In MIMO-IC, each BS communicates with only one user. In MIMO-IBC, each BS communicates with multiple users.

After significant research efforts in the past decades, the capacity region of the interference channel is still unknown. As a first order approximation to the sum capacity in high signal to noise ratio (SNR) regime, the degrees of freedom (DoF) have been extensively studied. Recently, interference alignment (IA) [1], [2] was shown to be able to achieve the information-theoretic maximum DoF of some interference networks. Moreover, it was shown that the overall DoF grows linearly with the cell number B for MIMO-IC and MIMO-IBC with some configurations [2], [3]. Nonetheless, in order to achieve the promised gain, either infinite symbol extensions over time/frequency [2] or a large number of antennas at each user are required [3], which is not realistic. In practice, the achievable DoF does not grow linearly with B in general [4].

Introducing relays to IC provides an effective way of reducing the number of symbol extensions. It was first noted in [5] that a relay-aided three-cell single-input-single-output (SISO)-IC can achieve a DoF of $\frac{3}{2}$ with only two symbol extensions. Similar results were obtained in [6] and [7] for B-cell MIMO-IC with M antennas at each BS and each user, where $\frac{BM}{2}$ DoF can be achieved with a half-duplex relay, or equivalently with two symbol extensions.

When relays are available in IC, on the other hand, it was found in [8] that another interference management technique called interference neutralization (IN) is essential for achieving high DoF. If there are more than one propagation paths from a source to its interfering destination, which is common in relay systems, multiple copies of one interference signal arriving at each user can add up to zero. In other words, the interference can be eliminated in the air, i.e., neutralized. Other techniques with the same idea in essence were independently proposed in the literature, albeit under different names such as distributed orthogonalization in [9] and orthogonalize-and-forward in [10].

The relay-aided IC represents a class of more complicated interference networks. With the possible exception of IA, interference avoiding and cancelation, IN as a new means of interference removal is still not well understood. To find the potential of relay-aided IC with constant coefficients, researchers have investigated the DoF for various settings. Analogous to IC and IBC, the achievable DoF of relay-aided IC with linear transceivers reflects the subspace dimension required to support interference-free transmission. This suggests that the maximum DoF achieved by linear transceivers can be found by analyzing

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the minimum numbers of antennas at the BSs, relays and users that guarantees the linear transceivers to be feasible. Such a feasibility analysis includes finding and proving the necessary and sufficient conditions.

For a relay-aided B-cell SISO-IC where each BS and each user respectively have a single antenna, a maximum of B non-interfering data streams can be transmitted with a single full-duplex B-antenna relay, or with $B(B-1)+1$ half-duplex single-antenna two-hop relays [10]. In [11], the authors considered the SISO-IC with multiple half-duplex relays where the direct links among BSs and users exist. They showed that by using $B(B-2)$ half-duplex relays each with a single antenna, a total of B data streams can be transmitted without interference. Compared with [10], the number of relays is reduced because the direct links are considered. In these scenarios, the interference management scheme for achieving the maximum DoF is pure IN (PIN), where only the relays are employed to eliminate inter-cell interference (ICI). For relay-aided B-cell MIMO-IC where each BS or each user has more antennas than its desired data streams, IA can be employed in conjunction with IN to achieve the maximum DoF. The authors of [12] and [13] introduced aligned interference neutralization for a two-source two-destination relay-aided MIMO-IC, respectively considering one instantaneous relay and two full-duplex relays. The achievable DoF region of the two-source two-destination two-relay aided MIMO-IC was further derived in [14]. For a relay-aided B-cell MIMO-IC where each BS conveys d desired data stream, the authors of [15] obtained a DoF upper bound. The results show that to transmit Bd data streams without interference, the total number of antennas at full-duplex relays needs to exceed Bd and a proper condition originally proposed for MIMO-IC in [16] needs to be satisfied.

While priori results for the achievable DoF in [11]–[14] and the DoF upper bound in [15] provide useful insights to understand the potential of relay-aided IC, for general multi-cell relay-aided MIMO-IC, the maximum achievable DoF remains

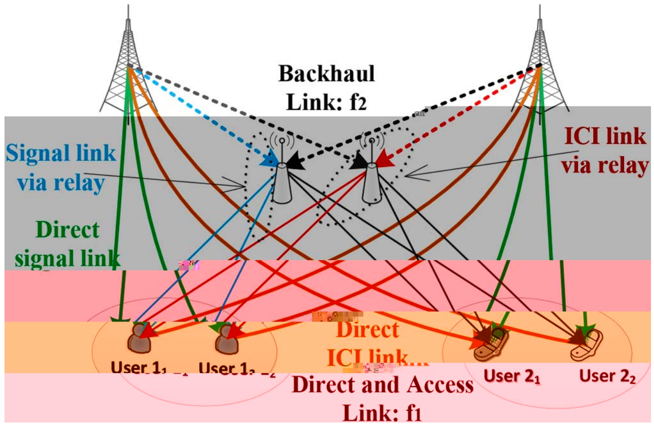


Fig. 1. Example of a two-cell relay-aided MIMO IBC with two users in each cell and two relays in the network, $B = 2, K = 2, N_R = 2$.

All elements in these channel matrices are independent and identically distributed (*i.i.d.*) random variables.

The received signal at user b_k can be expressed as

$$\begin{aligned}\boldsymbol{Y}_{b_k} &= \boldsymbol{U}_{b_k}^H\left(\boldsymbol{H}_{b_k,b}\boldsymbol{V}_{b_k} + \sum_{r=1}^{N_R}\boldsymbol{G}_{b_k,r}\boldsymbol{\Gamma}_r\boldsymbol{F}_{r,b}\boldsymbol{W}_{b_k}\right)\boldsymbol{x}_{b_k}\\
&\quad + \boldsymbol{U}_{b_k}^H\sum_{k'=1,k'\neq k}^{K}\left(\boldsymbol{H}_{b_k,b}\boldsymbol{V}_{b_{k'}}\right.\\
&\qquad\qquad \left. + \sum_{r=1}^{N_R}\boldsymbol{G}_{b_k,r}\boldsymbol{\Gamma}_r\boldsymbol{F}_{r,b}\boldsymbol{W}_{b_{k'}}\right)\boldsymbol{x}_{b_{k'}}\\
&\quad + \boldsymbol{U}_{b_k}^H\sum_{b'=1,b'\neq b}^{B}\left(\boldsymbol{H}_{b_k,b'}\boldsymbol{V}_{b'}\right.\\
&\qquad\qquad + \sum_{r=1}^{N_R}\boldsymbol{G}_{b_k,}\boldsymbol{\Gamma}\end{aligned}$$

As long as the term in the bracket is full rank, we can always design the outer transmit matrix $\boldsymbol{V}$

of IA feasibility. This indicates that the DoF upper bound derived from the proper condition may not be achievable.

To analyze the sufficient condition for the relay-aided MIMO-IBC, we linearize the cubic equations by considering two interference coordination strategies. This is one typical methodology to find and prove the sufficient condition [22].

Specifically, when the transmit matrix at each BS for the backhaul link and the receive matrix at each user (i.e., $\boldsymbol{W}_b^I$ and $\boldsymbol{U}_b^I$) do not participate in removing ICI, we call the strategy as the *coordinated interference neutralization (CIN)*.

When the ICI is only eliminated by relays with $\boldsymbol{\Gamma}_r$, we call the strategy as *pure interference neutralization (PIN)*.

In general, the achievable DoF derived from the sufficient condition of PIN feasibility is lower than that from CIN, and both are lower than that from the GIN feasibility. Nonetheless, as shown later, for a special class of systems with minimal antenna configuration, the sufficient condition of CIN feasibility coincides with the necessary condition of GIN feasibility, i.e., it is the necessary and sufficient condition for the GIN.

The following theorems respectively provide the feasibility conditions when considering the two strategies. When each BS has enough antennas to avoid all the ICI, i.e., overhead to obtain channels is also reduced. With the PIN, the achieved DoF is the lowest, yet the required channel information is also minimal. Therefore, the CIN and PIN strategies are of practical interest, despite that they are not optimal in the sense to achieving the maximum DoF of the considered interference network.

IV. Proof of the Main Results

A. *Proof of Theorem 1*

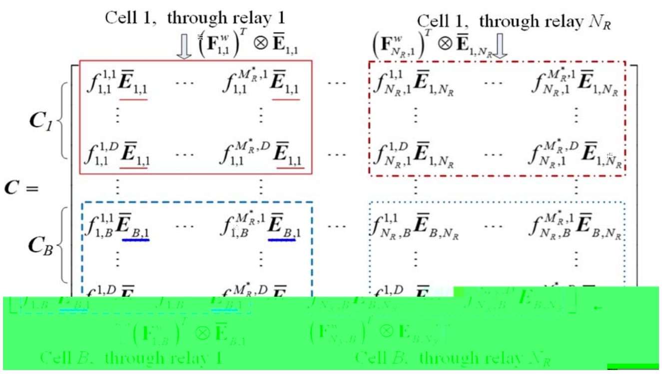
For the *coordinated interference neutralization*, the transmit matrix at each BS for the backhaul link and the receive matrix at each user are designed for other purpose instead of removing ICI. To analyze the feasibility condition of CIN, we set the inner transmit matrix $\boldsymbol{W}_b^I$ and the inner receive matrix $(\boldsymbol{U}_{b_k}^I)^H$ as arbitrary given matrices. Then, the ICI-free constraints (6) become linear equations of $\boldsymbol{V}_b^I$ and $\boldsymbol{\Gamma}_r$ as

$$\boldsymbol{H}_{b',b}^u \boldsymbol{V}_b^I + \sum_{r=1}^{N_R} \boldsymbol{G}_{b',r}^u \boldsymbol{\Gamma}_r \boldsymbol{F}_{r,b}^w = \boldsymbol{0}, \tag{14}$$

where $\boldsymbol{H}_{b',b}^u \triangleq (\boldsymbol{U}_b^I)^H \boldsymbol{H}_{b',b}, \boldsymbol{G}_{b',r}^u \triangleq (\boldsymbol{U}_b^I)^H \boldsymbol{G}_{b',r}$ and $\boldsymbol{F}_{r,b}^w \triangleq \boldsymbol{F}_{r,}$

In the sequel, we first construct the coefficient matrix as a block diagonal matrix by setting "0"s and "1"s in $\boldsymbol{F}^{w}_{r,b}$ and setting the elements in $\bar{\boldsymbol{E}}_{b,r}$ as arbitrary *i.i.d.* variables, and then construct each block diagonal matrix. Finally, we prove that the constructed coefficient matrix are full row rank with probability one.

We start by observing the structure of the coefficient matrix. We rewrite it in a more detailed form with $f^{i,j}_{r,b}$ denoting the effective channel coefficient from the jth antenna of BS b to the ith antenna of relay r, which includes B row blocks as follows



Unlike MIMO-IC, the coefficient matrix of the relay-aided MIMO-IBC is not a sparse matrix, and hence the method of finding the non-singular Jacobin matrix in [4] is not applicable.

The coefficient matrix $\boldsymbol{C}$ is composed of B blocks $\boldsymbol{C}_b = [\,(\boldsymbol{F}^{w}_{1,b})^T \otimes \bar{\boldsymbol{E}}_{b,1} \quad \cdots \quad (\boldsymbol{F}^{w}_{N_R,b})^T \otimes \bar{\boldsymbol{E}}_{b,N_R}\,], b = 1,\ldots,B$. The structure of matrix $\boldsymbol{C}$ suggests that if the non-zero columns in $(\boldsymbol{F}$

composed of $L-1$ block matrices each with size $J\times$

The compact form of IC[redacted]
b is therefore(

[redacted] $\boldsymbol{H}_b^{av} + \sum_{r=1}^{N_R}$ [redacted]

From the proof, the parameter n_1 can be viewed as the number of variables required from one relay antenna besides the $(L-1)M_R^*$ variables provided by the $L-1$ "private" antennas to eliminate the matrix composed of L statistically independent block matrices. This indicates that it is full row rank with probability one.

Type II: When $m > 1, j = 1$, the sub-matrix reduces to

$$\hat{\boldsymbol{C}}_b^{m,1} = \left[\begin{array}{ccc|c} \boldsymbol{P}_1 & & & \bar{\boldsymbol{E}}_{r_L^1} \\ \hline & \boldsymbol{P}_2 & & \bar{\boldsymbol{E}}_{r_L^1} \\ & & \ddots & \vdots \\ & & \blacksquare & \end{array}\right.$$

[redacted]

(A.13)

[redacted]b-matrix is constructed following *Rule 3*, the blocks
in the last (A.13)

APPENDIX B
PROOF OF LEMMA 2

Proof: Remind that matrix $\hat{\boldsymbol{C}}_b$ in (20) has D row blocks and R_B column blocks. Condition (11a) ensures that $\hat{\boldsymbol{C}}_b$ is a fat matrix, which is necessary to ensure it as full row rank.

An immediate way to construct $\hat{\boldsymbol{C}}_b$ is to set it as a type IV matrix shown in (22), where

Consequently, when the conditions in Theorem 1 are satisfied, the matrix $\hat{\boldsymbol{C}}_b$ can be composed as a block diagonal matrix whose diagonal matrices are full row rank type II matrices.

Case 1.3: $2 \leq \lfloor \frac{M_R^*}{n_1} \rfloor < \frac{D}{n_2} \leq \frac{M_R^*}{n_1}$.

In this case, we know that $\lfloor \frac{D}{n_2} \rfloor < \frac{D}{n_2} \leq \frac{M_R^*}{n_1}$. We also rewrite (B.2) as (B.6). Then, we need to find a subset $\Omega^0 = \{(m_1, j_1,$

By multiplying $\lceil \frac{D}{D-}$

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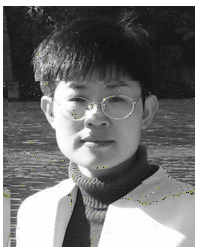

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